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PROSPECTIVE MATHEMATICS TEACHERS' TECHNOLOGY USAGES: A CASE FOR DYNAMIC GEOMETRY SOFTWARE

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Abstract: The purpose of the study is to introduce dynamic geometry software to prospective mathematics teachers and investigate their usage processes of technology on a geometric task. The participants of the study were three prospective mathematics teachers purposefully selected from 52 sophomores by their technological pedagogical content knowledge scores. A geometry task was given to each participant and their progresses on three dynamic geometry software were observed regarding their reasons for selection and usage. According to results, it was raised that technological pedagogical content knowledge of prospective teachers have great influences on purposes of using drag and measurement features of dynamic geometry software. The participants reached appropriate generalization using Geometry Expressions with its symbolic nature thus they stated it as easy to use while testing constructions or exploring geometrical constructs.

Key words: Dynamic geometry; Teacher education; Mathematics education; Dynamic geometry modalities; TPACK

1. Introduction

Since education forms a basis for both imagination and life, it is apparently inevitable to use technology in education (Santosh, 2013). In educational settings, since there may be different both technological and non-technological tools for demonstrating, teaching or learning a topic, needs for technological tools can emerge when someone knows them, chooses to use them and beliefs him or her capabilities to use them (Zbiek, Heid, Blume & Dick, 2007). However, every people resist to new changes in the beginning, as in their nature (Zbiek et al., 2007). Kaput (1992) stated this resistance as "major limitations of computer use in the coming decades are likely to be result of limited human imagination and constraints of old habits and social structures" (p.515). Pea (1987) defined technological tools for education as cognitive tools (CT). CT helps to go beyond of limitations of human mind (Pea, 1987).

In mathematics education, there are different learning tools and materials. Teachers can design both technical and conceptual learning activities by using these tools. Similarly, they can use the CT for procedural works, gaining intuition, discovering new patterns, exploring, proving and testing with their students (Zbiek et al., 2007). This construct of the CT is suitable for teaching geometry. Because, according to epistemological perspective of geometry education, geometry is considered as activities with at least two aspects; axiomatic nature of geometry without spatial experience and spatial foundations of geometry. (Laborde, Kynigos, & Strasser, 2006). The main ideas of these aspects are concepts and logical relationships about space, procedures and relations, which are used within society. In geometry education, these two aspects intersect. Especially, in middle school level, students generally need spatial figures or diagrams to construct knowledge about geometrical concepts (Laborde, Kynigos, & Strasser, 2006).

1. 1. Theoretical framework

One of the CT categories for geometry education is dynamic geometry software (DGS). DGS involve manipulation with geometric objects in both two-dimensional and three-dimensional geometry. A well-constructed figure, which is constructed by considering geometrical properties belong to one, in DGS is not just a static diagram; rather it is a variable and dynamic figure that can carry conceptual ideas (Leung, 2008). DGS provides opportunities students to construct and interact with this

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dynamically so they can examine many examples of the constructed object (Özçakır, 2013). By this way, students have the opportunity to experiment with the tool, so that the dynamic geometry software serves as a laboratory for mathematics classroom (Tabach, 2011). For example, GeoGebra, which is DGS, can create hot-links for mathematical objects to show their both symbolic representations and visual representations at the same time (Hohenwarter & Jones, 2007). Hot-links provide to see changes in an object with multiple representations of it at the same time. Therefore, students can manipulate the geometric objects and observe changes in multiple representations of the objects provided by hot-links and with real-time measures (Laborde, Kynigos, & Strasser, 2006; Özçakır, 2013). Hence, they can test, observe, record, manipulate and predict geometric objects, concepts, beliefs and theorems with DGS like as a laboratory experiment for mathematics (Forsythe, 2007; Hill & Hannafin, 2001). By this means, school geometry can become an experimental environment with DGS.

First DGS was developed in 80s. Some of the DGS are Cabri-Geometer, Geolog, Geometry Inventor, Geometric Supposer, CarMetal, Geometer's Sketchpad, Geometry Expressions and GeoGebra. These DGS can be differentiated for the purpose of use and their properties. While some of the DGS like Geometer's Sketchpad attempt to model the orthodox use of a compass and a straightedge in Euclidean geometry, others have additional features such as construction analytically like GeoGebra or symbolic representation like Geometry Expressions. On the other hand, all of them meet in some common features such as measuring capabilities and the possibility to drag objects about the screen, which offer new opportunities for school mathematics (Gonzales & Herbst, 2009). Different strategies or modalities for dragging have been studied (Arzello, Olivero, Paola & Robutti, 2002; Leung, 2008). Arzarello and others (2002) defined seven dragging modalities for working in dynamic geometry environments with dragging tools while trying to investigate conjecture making processes of students for a geometrical problem. These modalities are wandering dragging, guided dragging, bound dragging, dummy locus dragging, line dragging, linked dragging, and test dragging. Leung (2008) derived these modalities to structure a situated hierarchical scheme for students' works of geometrical conjecture. His lens for the model, which he described, is functions of variation (contrast, separation, generalization, and fusion) proposed in Marton, Runesson and Tsui's (2004) theory of learning and awareness. Leung (2008) described his model as that drag for contrast stands for wandering dragging to see differences without focusing to a specific object whereas drag for separation refers guided dragging or dummy locus dragging to inspect objects with focusing one or more of them (Leung, 2008). Drag for generalization is about facilitating generalization with drag for test by discerning or verifying invariants under varying appearances of the object of exploration and also checking the robustness of the idea constructed under DGS (Leung, 2008). When this check is achieved, drag for fusion is reached which is about integrating all previous modes with simultaneity so that students experience different features of an object at the same time (Leung, 2008). Besides these dragging modalities, DGS provides a representation of the result of measuring process in dynamic way so that usage purposes of these measurement tools could be varied.

Since a number or symbolic expression appears on screen for measuring act dynamically, students do not need to calculate directly in measuring process (Olivero & Robutti, 2008). Gonzales and Herbst (2009) stated measuring opportunities and dynamic features in DGS allow students to see relationships within geometric properties of objects rather than their shapes. In addition, dynamic measuring features within DGS provides students analyze and investigate deeper while they are working for a problem situation. Olivero and Robutti (2008) have proposed modalities in the case of measuring in two categories as "the shift from the spatio-graphical field to the theoretical field" and "the shift from the theoretical field to the spatio-graphical field". The shift from the spatio-graphical field to the theoretical field has occurred if students explore a geometric problem without having a clear idea in mind and using measurements towards a particular property of an object, indirectly. So that they may manage to formulate a conjecture by connecting measurements to graphical observation of the object (Oliver & Robutti, 2008). Three modalities were defined for this category. First of all, wandering measuring refers having no clear idea about the object regarding problem so exploring situation randomly (Olivero & Robutti, 2008). Secondly, guided measuring stands for doing structured exploration of the object and examining particular cases in order. Thirdly, perceptual measuring is defined as checking the validity of a perception with measurements while students have some ideas or intuitions about some properties of relationships on the object but they are not sure of their perceptions

(Olivero & Robutti, 2008). On the other hand, if students make reverse actions as using measurements to move from a conjecture, or a proof, of a discovered property back to the figure in order to validate, their purposes of measurement are described with the shift from the theoretical to the spatio-graphical field category (Oliver & Robutti, 2008). Two modalities were explained for this category. Firstly, validation measuring is about checking conjecture within DGS by doing measurements to either accept or refuse it after students formulated a conjecture. Secondly, proof measuring stands for an extension so that students may start over to use DGS after constructing a proof to understand the proof better and get a better explanation about it (Olivero & Robutti, 2008).

1. 2. Motivation for the study

Previous researches revealed that students have enthusiasm to learn while dealing with geometrical tasks in DGS (Furner & Marinas, 2007; Shafer, 2008). Moreover, using dynamic geometry and mathematics activities in lessons may enhance students' geometrical creativity and problem solving abilities (Battista, 2002; Güven, 2012; Laborde, Kynigos, & Strasser, 2006; Özçakır, 2013; Sinclair & Robutti, 2013). Although some technological tools can increase level of learning of students, teachers generally do not want to integrate new things in their curricula (Zbiek et al., 2007). This can be results of their fears, old habits, beliefs about themselves for applying and using effectively technology in lessons (Ertmer, 1999). In addition, integration technology into education requires content, pedagogy and technology knowledge for teachers, and integration of these components. Koehler and Mishra (2008) stated, "the development of [Technological Pedagogical Content Knowledge] TPACK by teachers is critical to effective teaching with technology" (p. 3). Because, TPACK refers to the knowledge of any learning content to teach with suitable pedagogy by using related technological tools (Koehler & Mishra, 2005). TPACK provides ability of interpreting the relationships between content, pedagogy and technology knowledge, and with TPACK, teachers are able to teach concepts by using appropriate pedagogical strategies and suitable technological tools (Schmidt, Baran, Thompson, Mishra, Koehler & Shin, 2009). For example, for the case of teaching a geometrical concept by using DGS teacher should understand how to illustrate or use these illustrations within DGS, should know and administrate appropriate pedagogical techniques needed to represent the concept in DGS, should deal with any challenges or benefit of using DGS, and should foresee that the DGS is more suitable for learning the concept than other tools (Yiğit, 2012).

De Villiers (2004) noted that if educators want to provide learning environment supported with DGS in schools, firstly pre-service and in-service mathematics teachers should use DGS effectively and should be engaged using dynamic geometry activities in learning and teaching phases. Similarly, Allison (1995) stated that providing technological opportunities into schools is not enough for successful integration of technology in education so that curriculum makers as well as teachers should find suitable ways to use technology effectively for teaching purposes. Therefore, the purpose of the current study is to introduce DGS to prospective mathematics teachers and investigate their progresses on dynamic geometry activities, and views about geometry instruction supported by DGS. As De Villiers's (2004) statement for DGS usage in schools, firstly teachers and prospective teachers have a knowledge about DGS, and should practice DGS by their own before administrating in class. In the light of these thoughts, in this study, it was investigated how prospective teachers engage with DGS. Because of rapid integration of technology into our life, teaching with and about new technologies has its place in education. Thus, guiding prospective teachers to discover their own TPACK with experiences is important as the development of pedagogical content knowledge (Niess, 2005). Therefore, in the current study, prospective teachers' TPACK for geometry is considered to investigate which level of TPACK affects their choices, efforts and persistence on given a geometry task with DGS, and their views about dynamic geometry activities. For this purpose, prospective mathematics teachers' interactions with GeoGebra, Geometers' Sketchpad and Geometry Expressions were inspected. In addition, this study contributes to the limited literature about TPACK studies related with usage dynamic geometry software in Turkey.

2. Method

This current study was designed as a case study in order to get deeper understanding about engagement and decisions of prospective mathematics teachers within DGS in learning environment. The case of this study was approaches of prospective mathematics teachers to a sample geometric problem with GeoGebra, Geometers' Sketchpad and Geometry Expressions software. In this study, this research question was answered; "How do prospective mathematics teachers approach and choose DGS while solving a geometric problem?".

2. 1. Selection of participants

The study composed of the purposeful sampling of three sophomores at Elementary Mathematics Education in a public university in Central Anatolia Region of Turkey. This sampling method is used to select information rich cases in-depth studies (Patton, 1990) and to take at least one person from each group based on their TPACK levels. Thus, there participants were selected from 52 sophomores who succeeded the course of Investigating Mathematical Concepts through Dynamic Geometry Software in the university in accordance with their TPACK scores. The participants have been chosen from three TPACK levels as low, moderate and high in order to investigate whether they have common or different progresses among different DGS tools.

At the beginning of the research, researcher administrated perceived TPACK regarding geometry scale developed by Bulut (2012). This scales included 51 items in seven dimensions as Technological Knowledge, Pedagogical Knowledge, Content Knowledge, Technological Pedogeological Knowledge, Technological Content Knowledge, Pedagogical Content Knowledge and TPACK. This scale was specifically related to geometry education and TPACK contents so that prospective teachers' TPACK levels regarding geometry were assessed with it in this study. This scale was designed as 6-point Likert scale ranging from strongly disagree to strongly agree. Students' placements for TPACK levels according to their scores in this scale were presented in Table 1.

Table 1. *Students' TPACK levels according to their scores*

Levels	Ranges of scores	f
Low	1,00 – 2,66	12
Moderate	2,67 – 4,33	24
High	4,34 – 6,00	16

2. 2. Procedure

The current study was administrated prospective mathematics teachers who had completed a course about *Investigating Mathematical Concepts through Dynamic Geometry Software*, which was an elective course of the department of Mathematics Education at the university. In this course, some constructs about interactive mathematics, computer assisted learning and geometrical figures and drawings were introduced to students. The researcher was instructor of this course and this course was lasted 15 weeks with two hours a week. The course was planned as involving two parts. The first part was administrated in a classroom environment and the second part was administrated in a computer laboratory. In the first part, theoretical contents included interactive mathematics, computer assisted learning, and geometrical figures and drawings were taught. In the second part, practical contents included introduction to DGS, and dynamic geometry activities with these DGS were performed. At end of the course, students were asked to create new dynamic geometry activities in lessons.

In this study, a geometric problem was presented to prospective mathematics teachers in worksheets. Researcher investigated their progresses on the task. Each participant studied on the task individually. The participants were asked to deal with this task by using all three DGS they learned in their ways. Additionally, they were free to choose with which DGS they wish to start. The problem which was given to the participants was "*sequence of tangent circles"* related with mathematical induction (Figure 1) from Lyublinskaya and Funsch's (2012) study. Thereby, preservice teachers asked to construct a generalization with mathematical induction by using dynamic geometry.

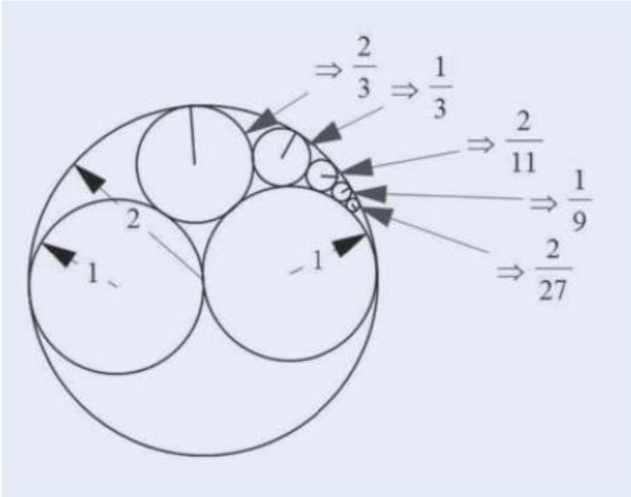


Figure 1. *Constructing sequence of tangent circles (Lyublinskaya & Funsch, 2012)*

In the problem, it was asked to students: "Supposed that there are two circles of radius 1 tangent to a circle of radius 2 and each other. Another circles of radius smaller than 1 are constructed tangent to outer circle, one of the circle of radius 1 and the last constructed circle. What is the nth circle radius in this sequence of circles?".

2. 3. Data collection and analysis

In this study, task based interviews, observations, screen shots of the task process in DGS, and participants' worksheets have been used to gather data. In task based interview, researcher asked questions like "*How do you construct this figure?, Can you explain why did you do this?",* and *"What do you think in this process?*" which provide to investigate prospective mathematics teachers' process during the task in detailed. The data were analyzed through one of the qualitative data analysis methods, content analysis. The data had been transcribed and analyzed by the researcher and two experts who had doctoral degrees in the field of mathematics education. Codes were asserted from regarded categories and themes expressed in the theoretical framework of this study (Table 2).

Table 2. *Codes, categories and themes for data analysis*

Themes	Categories	Codes
Dragging	Drag for contrast (DC)	Random dragging
		Disorganized comparison
	Drag for separation (DS)	Guided dragging
		Organized comparison
		Dragging with focusing
	Drag for generalization (DG)	Dragging to test
		Dragging to conjecture
		Generalization with visual info
	Drag for fusion (DF)	Using multiple tools
Measuring	Wandering measuring (WM)	Random dragging to measure
		Disorganized measuring
	Guided measuring (GM)	Guided dragging to measure
		Organized measuring
	Perceptual measuring (PEM)	Dragging to check perception
		Measuring to conjecture
	Validation measuring (VM)	Measuring to check conjecture
	Proof measuring (PRM)	Measuring to proof
		Measuring after completion

The abbreviations in this table were used as a tag in findings section in order to clarify data. In the data analysis process, data reduction, data display, drawing conclusion and verification steps has been utilized.

3. Results

In this study, prospective mathematics teachers' usage of dynamic geometry processes has been investigated in Geometry Expressions, GeoGebra and Geometer's Sketchpad with a sample geometric problem. Students' progresses on DGS were asserted regarding dragging and measuring modalities for DGS. Students were coded as L, M and H in order to preserve their anonymity. Student L had low level of TPACK, student M had moderate level of TPACK and student L had low level of TPACK.

3. 1. Progresses of student H in dynamic geometry environment

Student H used Geometry Expressions as first tool for the task. She started her construction with four random circles and she dragged three of them in a big one [DS]. Then she used constrain tool for measures in Geometry Expressions to adjust tangent properties and radius as *r, r* and *2r* [GM]. After that she continued to construct other tiny circles regarded problem situation on the task similarly. Therefore in construction process, she followed a route for construction random circles, dragging them appropriate locations [DS] and constraining measures and locations of them [GM]. It was observed that she has comprehensive knowledge of using construction and constraining tools of Geometry Expressions in order to construct circles and to constrain properties of circles that required for the task, while working on the task (Figure 2).

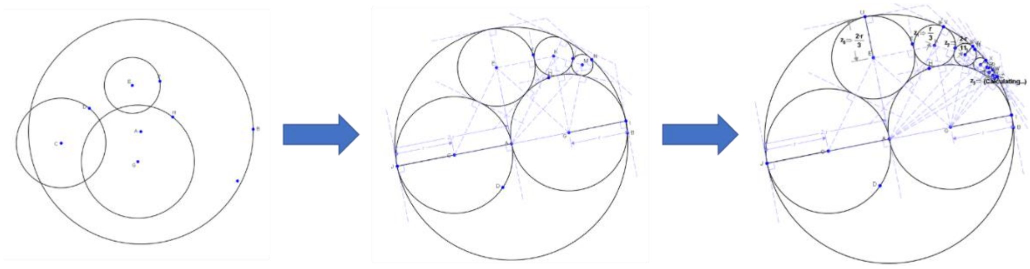


Figure 2. *Task process of student H with Geometry Expression*

After constructed some circles, she continued to with measurement tools of Geometry Expressions and measured radii of circles in numerical forms [GM] and also in symbolic forms [PEM]. In task-based interview sessions these measurement decisions asked while she was performing her work. She stated that she, firstly, wanted to know measurement in numerical form and then she made measurements in symbolic forms to discover and analyze relationships between radii of circles with proportional to r which is the radius of one of the first inner two circles. However, in task based interview sessions, while discriminating relationships between radii for all constructed circle, geometry expressions needed more time to calculate measures in symbolic forms. She stated this obstacle as; "…calculation [the measures of radii in symbolic forms] took too much time and even after waiting [nearly] much time, it has still continued the calculation. I have only got [measures of] radii of three circles…" (see Figure 2). Because of this delay in process, she used these three calculated radii and given radius of initial inner circles, which was *r*, to generalize and find a general formula for the measure of radius of n^{th} circle as $\frac{2r}{n^2+2}$ by accepting *n=1* which is the circle that tangent to first three circles. After writing down this formula for nth circle of radius, she had verified her conjectures about the formula with Geometry Expressions by validating measures of radius for circles with numerical measurement tool [VM]. She, as a final step with this DGS, started over to reconstruct up to n=2 circle by calculating their radii with her formula and constrained measures of them regarded her calculation to validate her formula [PRM] (see Figure 3). She moved on with GeoGebra to work on task.

GeoGebra and Geometers' SketchPad processes of her were nearly similar in terms of construction processes. She used final output of her works within Geometry Expressions as a reference tool for constructing process in both GeoGebra and Geometers' SketchPad [PRM]. She explained reason of this decision in task-based interview sessions as "*GeoGebra and Geometers' SketchPad are similar programs. While working on these DGS, I need to compute which value [of radii] are needed to make tangent these circles which I can find easily with Geometry Expressions. I do not need to compute exact values for radii of all circles with Geometry Expressions, since it has constraint tool and with this tool I can attend properties even after drawing figures [GM].*" Therefore, according to this transcript, she fused descriptions and measures acquired using different tools of Geometry Expressions with construction processes within GeoGebra and Geometers' SketchPad. She used multiple tools in Geometry Expressions for dragging and measuring to get related measures to help her constructions [DF] within other DGS. These values for radii calculated in Geometry Expressions helped her construct circles by fixing radii of circles in GeoGebra and Geometers' SketchPad's constructions. As a starting point of construction process within GeoGebra, she constructed the outer circle with a fixed radius of 2-unit by circle with radius tool [GM]. She continued with constructing a line through radius of these circle and determined possible centers of two inner circles by measuring two center points on two half of diameter of the outer circle [GM]. Then she constructed two inner circles with radius of 1-unit from these center points. After that she constructed two line segments to find out location of following circle. Then she performed similar circle construction processes. After she completed her constructions, she constructed some points at intersection of circles. Then she made some measurements of these points to centers for radii and stated that she used these measurements in order to check her assumptions about whether these circles were tangent to three of others [PEM] and also tried to drag all points to test her construction [DG] (Figure 3).

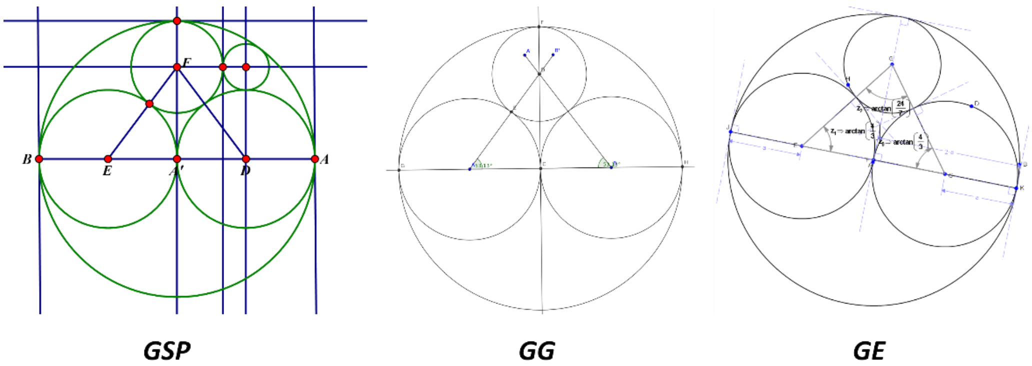


Figure 3. *A part of student H's works on Geometer's Sketchpad (left) and GeoGebra (middle) based on output of Geometry Expressions (right)*

Her works in Geometers' SketchPad were similar to one in GeoGebra except one difference. In Geometers' SketchPad, she moved in a route for constructing using measurements gathered from Geometry Expressions [GM], checking her assumptions with measurements [PEM] and testing her drawing by trying to drag all points whether her construction broke down [DG]. Although she used values of measurements in Geometry Expressions while constructing circles for problem situation on Geometers' SketchPad, she used dragging tool to place circles after third circle [DS] an approximate location to be tangent to other three circle. Therefore, she failed to construct circle in correct location. She only dragged circle to where they could be. To summary, she made her generalization in Geometry Expressions and test this generalization with GeoGebra and Geometers' SketchPad.

3. 2. Progresses of student M in dynamic geometry environment

Similar to student H, student M used Geometry Expressions as first tool for the task. At the beginning of her work, she drew the first three circles as random, in other words, she tried to draw and drag circles as near tangent state to each other [DS]. After that she used constraint tool of Geometry

Expressions to make tangent these circles to each other and to fix their measures of radius as *a*, *a* and *2a* [GM] (Figure 4).

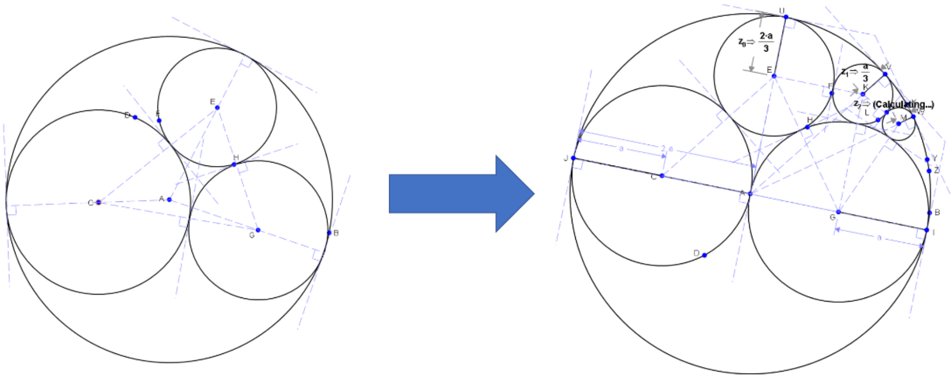
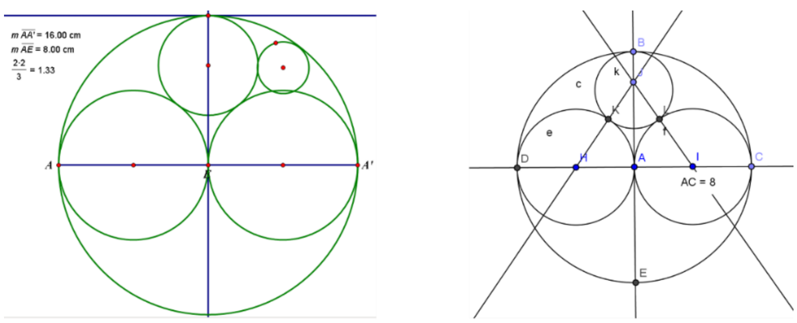


Figure 4. *Task Process of the student M with Geometry Expressions*

She also used outputs of distance/length tool to discriminate relationships between radii [GM] and tried to reach a general formula by generalizing these outputs [PEM]. In the meanwhile, she also faced with the same obstacle that student H had encountered (*see Figure 4*). M stated this situation as; "*I think, Geometry Expression either has frozen or needs more time to calculate. I am starting to derive these already calculated measures into a general formula. I think, these measures will be enough to find a general formula [PEM]*". Thus, she took as reference this pattern and these symbolic measures in order to find a general formula [PEM]. She generalized these symbolic measures as $r = \frac{2a}{2+(n-1)^2}$ by stating *n=1 wa*s the one of the initial inner tangent circles which have equal measure of radius of *a*. After that, she made a quick check of her conjecture about formula with Geometry Expressions by measuring each radius with numerical measurement tool [VM]. When she completed her work with Geometry Expressions, she moved on with GeoGebra as a tool for task.

Student M had also tried to use outputs of her work in Geometry Expressions similar to the student H. However, in contrast with the student H, the student M benefited from visual clues on output like triangle between centers of circles and also proportion between radii of these circles. Therefore, she also fused descriptions in Geometry Expressions with the figures across platforms. Before starting the task with GeoGebra, she measured angles of triangle in Geometry Expressions [GM]. She started task by constructing this triangle by using the measures of its angle and used the corners of it to determine centers of circles [DS & GM]. After that, she used these points as centers and constructed initial inner circles for the task. After that, she constructed outer circle by measuring and locating tangential points of these circles to outer one [PEM] (Figure 5). Moreover, she dragged some points on construction to check whether tangential of circles failed or not [DG].



GSP GG

Figure 5. *Parts of student M's works on Geometer's Sketchpad (left) and GeoGebra (right)*

However, she could not make any further attempts from these circles. Then, she dropped the task and could not construct other circles to validate her assumption about formula generated in Geometry Expressions. In Geometers' SketchPad, she followed different strategy. She applied construction steps like compass-and-straightedge construction for given circles in the task statement and used dragging feature of Geometers' SketchPad to make other circles tangent to each other [DS] (*see Figure 5*). However, as seen in figure 5, she did not accomplish to construct circles tangent to each other. While dragging some points of circle she noticed that these circles were only seem tangent to each other [DG]. Hence, her measures of radii did not represent any pattern related with her formula [PEM]. Therefore, her process in Geometers' SketchPad could not even demonstrate clues for relationships between figures.

3. 3. Progresses of student L in dynamic geometry environment

Student L followed different path from others and he chose GeoGebra as a starting tool for his work. In his task processes, he made nearly same processes of works with either GeoGebra or Geometers' SketchPad. With these DGS, he, firstly, constructed a line segment and determined middle point of it [GM]. After that he determined quarter points of this segment [GM]. When it was asked "*Why did you construct this line segment?*" he replied as "*Because I need two tangent circles and they defined as equivalent circles*". He constructed circles by using marked points on this line segment such as middle point of line segment for outer circle and quarter points for inner tangent circles. His construction process was followed by dragging to a new circle to make tangent them as asked in the task statement [DS]. However, his processes in GeoGebra and Geometers' SketchPad were not helpful to clearly adjust tangential properties of these circles since the circles were dragged to be seem as tangent but not tangent to each other. Moreover, since circles, except constructed on middle and quarter points on same line segments, were not constructed properly, he found irrelevant measures of radii for these circles [GM] so that he could not form any pattern with these measures. Hence, he dropped working for task before making any generalization or assumption for radius of nth circle (Figure 6).

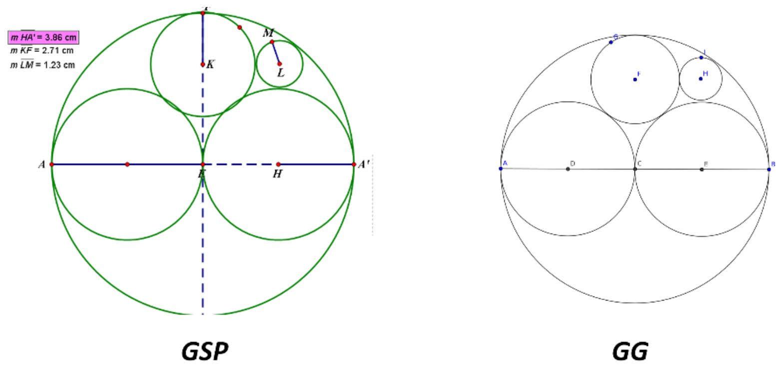


Figure 6. *Part of task Processes of the student L with Geometer's Sketchpad (left) and GeoGebra (right)*

In Geometry Expressions, he did not use any constraint tool to construct circles. Therefore, construction steps nearly similar to construction steps like compass-and-straightedge construction as in his processes in Geometers' SketchPad and GeoGebra. In other words, he constructed outer circle and measure quarter points on it diameter [GM]. Then he constructed other two inner tangent circles by using these points which he found in early step [GM]. He did not use Geometry Expressions features of constraint and tried to construct other circles with circle tool and drag them to approximately tangential locations [DS]. After constructed some circles, he tried to measure radii of these circles and became confused in this process since Geometry Expressions has two different measurement tools, symbolic and real. He tried to measure radii by using symbolic measure tool of GE. Therefore, outputs of measurement had meaningless computations [WM] (Figure 7). Because of these meaningless measures, he could not make any further attempts to generalize these results.

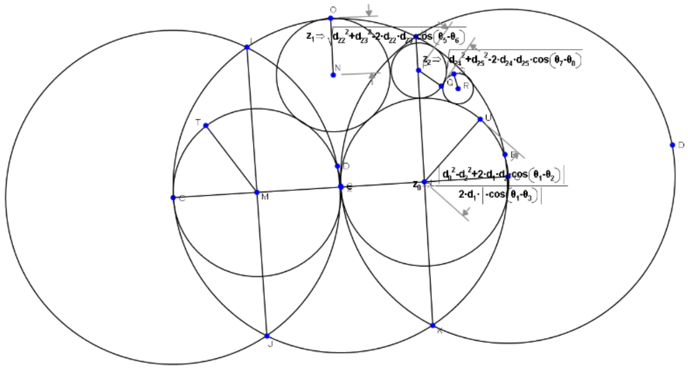


Figure 7. *Task process of the student L with Geometry Expressions*

To sum up, students H and M made proper and valid formulas with their works in Geometry Expressions in order to calculate radius of nth circle and successfully applied these formulas with GeoGebra and Geometers' SketchPad. In addition, both of them fused symbolic measures or visual representations at final outputs in Geometry Expressions to their works while constructing figures in other platforms such as GeoGebra and Geometers' SketchPad. Both of them used Geometry Expressions as a support tool for making construction on other DGS. However, the student L only showed some evidence for using either mathematical features or tools of software for all DGS. He followed different ways from the students M and H. While the M and the H can interrelate these all DGS, the L cannot see their relations with each other except relations between GeoGebra and Geometers' SketchPad. (Figure 8).

3. 4. DGS selection reasons

In the task processes, it was observed that the students M and H started task by using Geometry Expressions, and after completed the task with Geometry Expressions they continued task with GeoGebra and Geometers' SketchPad. However, the student L started task with GeoGebra and continued with other DGS. Students H described her reasons about choices of programs in task-based interview sessions as follows:

> *Researcher (R): Why do you choose Geometry Expression as a first tool?*
>
> *(H): Both, GeoGebra and Geometer's Sketchpad give numeric results for measures, but Geometry Expressions is capable to show measure not only in numeric form and but also in symbolic form.*
>
> *(R): You say reason for this choice is the difference between representation forms, why this form of representation [symbolic] is important for you?*
>
> *(H): Since, in this task, it was asked to find a general formula in these series of circles, I can use these symbolic measures to derive a general* formula, easily.

Hence, she made her choice for DGS by considering problem situation and requirements for the solution. It was observed that she have comprehensive knowledge of using all three DGS and their construction tools. The students M also stated her reasons for selection of program as follows:

> *(R): Why do you start the task with Geometry Expression?*
>
> *(M): Usage of this software is more appropriate to find a general formula, since it gives symbolic measures.*
>
> *(R): What do you think about how these symbolic measures affect your work?*
>
> *(M): I could translate these symbolic measure to a general formula as a function of radius of first circle.*

In the study, it was observed that she had knowledge to use Geometry Expressions properly. She could also use GeoGebra and Geometers' SketchPad fluently. Student L (L) followed different route from both the student H and the student M. He started the task with GG. When it is asked why he started with GeoGebra in the task based interview sessions;

> *(L): In lesson, we firstly learned and used GeoGebra, after Geometer's Sketchpad, and then Geometry Expressions. So, I am doing this now".*
>
> *(R): Can you describe differences between these programs?*
>
> *(L): GeoGebra and Geometer's Sketchpad are nearly similar programs. Geometry Expressions is, I think, different from them.*
>
> *(R): What makes GeoGebra and Geometer's Sketchpad similar and Geometry Expressions different from them?*
>
> *(L): These two have allow us to construct geometric figures like construct on paper, I mean, we can use compass-and-straightedge construction ruler within it. In the other one we can get symbolic results compared to these two. And construction steps are different from these two, but I cannot remember, now, how different constructing figures with this program.*

As seen on these transcripts, this student had insufficient knowledge of usage of Geometry Expressions. Moreover, it can be also deduced that this student knew usage of GeoGebra and Geometers' SketchPad. However, he did not have proper knowledge of usage of their all construction tools.

4. Discussion

Many studies revealed that DGS enhance students' performance on geometrical concepts (Forsythe, 2007; Gonzales & Herbst, 2009; Güven, 2012; Leung, 2008; Olivero & Robutti, 2008; Todd, Lyublinskaya & Ryzhik, 2010). In addition, DGS does not only aid in finding the sufficient and necessary condition, but also provides the information on which the proof was based. Moreover, in problem solving sessions DGS fosters verification process of students. This research guided and enlightened value of TPACK in terms of choosing and using suitable technological for dedicated tasks in learning settings via prospective teachers' works and how their approaches can change for same task in three dynamic geometry software. In this study, there prospective teachers were considered as representative of their classmates in three levels of TPACK. Their progresses in a geometry task with GeoGebra, Geometers' SketchPad and Geometry Expressions were reported in regard to drag and measure modalities of DGS.

Technology decisions of the prospective mathematics teachers regarding their TPACK levels were asserted from their works. The prospective mathematics teachers with moderate and high levels of TPACK perceptions successfully reached some solutions for the problem with the Geometry Expressions. However, the other prospective teacher with low level of TPACK perception was not successful in his solving processes for the problem neither with any DGS. First of all, since TPACK perception is related with knowledge of teachers, or prospective teachers for this study, about technology, pedagogy, content, and their relationships with each other, teachers or prospective teachers with moderate or high level of TPACK perception should be supposed to know related technology for a task as well as usage of it for learning situations (Yiğit, 2012). Therefore, the prospective teachers with moderate and high levels of TPACK perceptions made their decisions for related software successfully, in this study. Since they started to work with Geometry Expressions for the given problem, which included a generalization situation, they reached a general formula. Because, the Geometry Expressions provides symbolic representations for inspected geometric objects with dynamic representations. Therefore, this software could be considered as the ideal medium for making generalization of a situation in symbolic ways (Lyublinskaya & Funsch, 2012). In contrast to these prospective teachers, the other prospective teacher, who had low level of TPACK perception, did not make decisions for suitable technology for a generalization task and his works showed that his knowledge for usage of GeoGebra, Geometers' SketchPad and Geometry Expressions was observed as insufficient to complete the task in this study since along side of his decisions about DGS his works had been not completed. As stated by Koehler and Mishra (2005) TPACK perception of a teacher or prospective teacher affects his/her knowledge for deciding related pedagogy and technology for intended content. Hence, in this study, the prospective teachers with moderate and high levels of TPACK perceptions made wise decisions and meaningful works, hence, these prospective teachers had required knowledge to decide suitable technology for task and its usage (Schmidt et al., 2009) while other one was in failure. Symbolic geometry systems like Geometry Expressions can be used in formulating geometrical proofs and as the bridge between components of geometric and algebraic properties of geometrical figures. In addition, GeoGebra and Geometers' SketchPad can be used to form a conjecture, they can give little help while finding a symbolic and general formula for a geometrical relation (Todd, Lyublinskaya & Ryzhik, 2010). In this study, it was seen that Geometry Expressions provided students useful tools to construct figures easily by making constraints and to measure symbolically to notice the value or the formula of parameters. This feature of Geometry Expressions can assist the creation of conjectures (Lyublinskaya & Funsch, 2012).

The prospective mathematics teachers' works have clues for their solutions processes for given geometric problem. Differences in prospective teachers TPACK levels can have an influence in choosing strategies and using of DGS suitable to given problem situation (Meng & Sam, 2013). In this study, prospective teachers' works have some changes in terms of drag and measure modalities regarded their levels of TPACK perceptions and different DGS. While working with Geometry Expressions, the prospective teachers with moderate and high levels of TPACK perceptions had nearly similar works for drag modalities. Both of them, made guided and focused dragging actions while dealing with problem situation. These actions about dragging for separation resulted with meaningful drag as stated by Leung (2008) and these actions showed that these prospective teachers, with moderate and high TPACK perceptions, had understood the problem situation and made conscious decisions for solving. However, the low TPACK level prospective teacher could not make any drag acts specifically listed in any modalities. He just constructed and make measurements with this DGS. On the other hand, he was observed as dragging some constructed figures and points by focusing on certain things with GeoGebra and Geometers' SketchPad. Similar to other prospective teachers, he showed signs for drag for separation (Leung, 2008). Additionally, the high TPACK level prospective teacher had dragged some critical points of the constructed figures to analyze and inspect changes in measures and figures while transferring her works in Geometry Expressions to other DGS which she used after this program. Therefore, she observed critical features at same time with drag for fusion (Arzarello et al., 1995; Leung, 2008). This construct only applied by this prospective teacher so that her TPACK perception so on her knowledge about geometric features of this application could affect her acts. Moreover, these prospective teachers made drag for generalization acts in their works in GeoGebra and Geometers' SketchPad as distinct from Geometry Expressions. They dragged some points of figures to test their constructions, therefore, they made these drags to make and validate generalizations with visual data since GeoGebra and Geometers' SketchPad do not allow users make explorations with symbolic representations in contrast to Geometry Expressions (Lyublinskaya & Funsch, 2012). Similar situation exists in the context of measurement with DGS. Prospective teachers made measurements while working DGS for the problem situation. First of all, similar to drag acts, the moderate and higher TPACK level prospective teachers used measurement tools of Geometry Expressions, GeoGebra and Geometers' SketchPad with similar purposes. They made guided and perceptual measuring in all three DGS and validation measuring in Geometry Expression as in same way. Additionally, the higher TPACK level prospective teacher also used measurement for proof purposes also. The other prospective teacher with low level of TPACK carried out wandering and guided measuring in DGS. However, his actions did not give any meaningful results. According to results and discussion written above, as TPACK levels get higher of prospective teachers, dragging and measuring modalities become complicated and more meaningful acts. Their knowledge for using and deciding related technology with related pedagogical elements for intended topic, namely TPACK, could make this difference.

In summary, it was seen that development of TPACK is also an important aspect of knowing how to use technology for reaching intended curriculum goals for the context of DGS. According to findings, prospective teachers' choice and strategies of using DGS suitable for given task are related with their TPACK perceptions. If prospective teachers know how to use appropriate technologies for teaching intended concepts, they can support learning of their future students in their professional life (Niess, 2005). Additionally, The strategic use of technology empowers mathematics teaching and learning processes (Dick & Hollebrands, 2011). Therefore, prospective teachers should have enough experiences with technological tools to understand capacities and features of them in educational settings. In geometry education, combination of technology, pedagogy and mathematics may reveal benefits of using DGS in learning environments. In order to incorporate technology in mathematics learning, mathematics educators should choose the best software in accordance with their knowledge of technology, pedagogy and mathematics as well as reinforcing understandings of their students.

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